EXPLICIT AND HIDDEN SYMMETRIES IN QUANTUM DOTS AND QUANTUM LADDERS

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Abstract The concept of dynamical and hidden symmetries in quantum dots and quantum ladders is introduced and developed. These symmetries are manifested in tunneling processes. If one studies the excitations in a given charge sector of nanoobject, then only the spin variables and/or electron-hole pairs are involved in the excitation spectrum. The spins in individual rung of a quantum ladder (QL) or in isolated complex quantum dot (CQD) form certain multiplets characterized by usual $SU(2)$ symmetry. This symmetry is broken due to spin transfer through QL or due to electron cotunneling through CQD. We show that dynamical symmetries of spin multiplets are unveiled in these processes. These symmetries are described by $SO(n)$ or $SU(n)$ groups in various conditions. We develop mathematical tools (fermionization procedure) for description of dynamical symmetries. The families of effective spin Hamiltonians of CQD and QL are derived in terms of generators of dynamical groups, and specific properties like Kondo tunneling through CQD and Haldane gap formation in QL are discussed.

Keywords: Spin ladders, quantum dots, Kondo tunneling, spin gap, excitons

1. Introductory notes

The symmetry of low-dimensional systems is a key to their peculiar properties [1]. It predetermines their thermodynamics, response to external fields, transport properties, phase diagrams, etc. As a rule, description of strongly interacting electrons in these systems should be constructed on non-commutative algebras, and specific structure of these algebras have direct consequences for observable physical properties of nanoobjects. In many physically interesting

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cases not only the symmetry of a given Hamiltonian but also the *dynamical symmetry* of low-energy excitations is relevant. Let us consider a system with Hamiltonian $\mathcal{H}_0$ whose eigenstates $|\Lambda\rangle = |M\mu\rangle$ form a basis to an irredicible representation of some Lie group G (μ numerates the lines of this representation). It is convenient to express the generators of Lie algebras via Hubbard operators $X^{\Lambda\Lambda'} = |\Lambda\rangle\langle\Lambda'|$. Then the Hamiltonian under consideration is expressed in terms of diagonal Hubbard operators

$$\mathcal{H}_0 = \sum_{\Lambda=M\mu} E_\Lambda |\Lambda\rangle\langle\Lambda| = \sum_\Lambda E_M X^{\Lambda\Lambda}\,, \tag{1}$$

so that

$$[X^{\Lambda\Lambda'}, \mathcal{H}_0] = -(E_M - E_{M'})X^{\Lambda\Lambda'}. \tag{2}$$

Then the symmetry group of the Hamiltonian is generated by the operators $X^{M\mu,M\mu'}$, which commute with $\mathcal{H}_0$, whereas the dynamical symmetry of $\mathcal{H}_0$ is generated by the whole set of operators $\{X\}$. This dynamical symmetry may be exposed, when $\mathcal{H}_0$ describes a quantum object, which is a part of larger system with the Hamiltonian $\mathcal{H}$, and its symmetry is violated by interaction with this environment. If the interaction scale is characterized by some energy $\mathcal{E}$, than the dynamical symmetry is determined by transitions between those states from the manifold E_Λ, which fall into the interval $\mathcal{E}$. We divide the Hubbard operators acting within this low-energy interval into subsets $\{S\}$ and $\{R\}$. Here S-operators generate the symmetry group G, whereas S- and R-operators together generate the dynamical group D. In this paper we study spin properties of quantum dots and quantum ladders, so the group G is in fact $SU(2)$ group of a spin momentum. It will be shown that the dynamical symmetry of this object is that of $SO(n)$ group. We will construct the corresponding algebras by means of Hubbard operators, rewrite Hamiltonians $\mathcal{H}$ in terms of group generators, discuss the possible ways of fermionization of these Hamiltonians and consider some specific properties of quantum dots and quantum ladders.

2. From spin rotator to Kondo tunneling

The symmetry of spin rotator is an intrinsic property of many low-dimensional spin systems. As was shown in [2], this symmetry predetermines the low-energy dynamics of zero-D quantum dots with even occupation in a tunneling contact with metallic Fermi resevoirs. Let us consider a double quantum dot (DQD)occupied by two electrons in a neutral state in a T-shaped parallel geometry (Fig.1) as a representative example.

In this geometry two valleys of DQD are coupled by tunneling V. In the limit of strong Coulomb blockade Q, such that $V \ll Q$, the energy spectrum of isolated DQD consists of ground state singlet with energy E_S, spin triplet with the energy E_T separated by the exchange gap $\delta = 2V^2/Q$ from E_S and

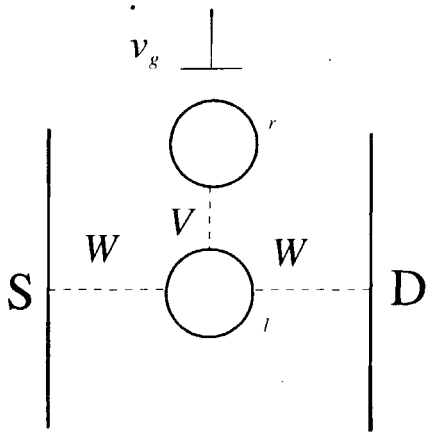

Figure 1. Parallel Double Quantum Dots in contact with source (S) and drain (D) metallic leads. V and W are tunneling coupling constants, v_g is a gate voltage.

two charge transfer excitons with large excitation energies $\sim Q$, the charging energy for a given well of DQD. Thus the indices Λ in the Hamiltonian $\mathcal{H}_0$ (1) acquire the values $\Lambda = S, T\mu$ with $\mu = 1, 0, \bar{1}$ standing for three projections of spin one.

The dynamical symmetry of the $\{S, T\}$ manifold is that of $SO(4)$ group. Two vectors generating this group are constructed by means of Hubbard operators (2) in the following way:

$$\begin{aligned} S^+ &= \sqrt{2}\left(X^{10} + X^{0\bar{1}}\right), \ \S_z = X^{11} - X^{\bar{1}\bar{1}}. \\ R^+ &= \sqrt{2}\left(X^{1S} - X^{S\bar{1}}\right), \ R_z = -\left(X^{0S} + X^{S0}\right). \end{aligned} \tag{3}$$

Here $\mathbf{S}$ is the spin 1 operator, while $\mathbf{R}$ is the R-operator describing S/T transitions. The spin algebra is o_4, which is characterized by the commutation relations

$$[S_\alpha, S_\beta] = ie_{\alpha\beta\gamma}S_\gamma, \ [R_\alpha, R_\beta] = ie_{\alpha\beta\gamma}S_\gamma, \ [R_\alpha, S_\beta] = ie_{\alpha\beta\gamma}R_\gamma \tag{4}$$

(α, β, γ are Cartesian coordinates, $e_{\alpha\beta\gamma}$ is a Levi-Civita tensor). These vectors are orthogonal, $\mathbf{S} \cdot \mathbf{R} = 0$, the Casimir operator is $\mathbf{S}^2 + \mathbf{R}^2 = 3$.

A gate voltage v_g applied to DQD turns the level positions essentially asymmetric, and the charging energy Q may be nearly compensated for at least one charge transfer singlet exciton (say, right, $\Lambda = E_r$). In this case we encounter a "Coulomb resonance" excitation, where the spin singlet and charge transfer exciton (also singlet!) are strongly intermixed, but the spin triplet is untouched by this resonance tunneling. This means that we deal with a manifold $\{S, T, E_r\}$. Then one more R-vector $\mathbf{R_1}$ and a scalar A should be included in the set of group generators. These generators are expressed in terms of Hubbard operators

as follows:

$$R_1^+ = \sqrt{2}\left(X^{1E_r} - X^{E_r 1}\right), \quad R_{1z} = -\left(X^{0E_r} + X^{E_r 0}\right),$$
$$A = i(X^{SE_r} - X^{E_r S}). \tag{5}$$

To close the alebra the commutation relations (4) which are valid also for $R_{1\alpha}$ should be completed by

$$[R_{l\alpha}, R_{1\beta}] = i\delta_{\alpha\beta}A, [R_{1\alpha}, A] = iR_{l\alpha}, \tag{6}$$
$$[A, R_{l\alpha}] = iR_{1\alpha}, \quad [A, S_{l\alpha}] = 0.$$

The system of commutation relations (4), (6) is that of o_5 algebra, and the manifold $\{S, T, E_r\}$ obeys $SO(5)$ dynamical symmetry, provided all three levels are involved in interaction in a framework of the Hamiltonian H. The Casimir operator for $SO(5)$ group is $\mathbf{S}^2 + \mathbf{R}^2 + \mathbf{R_1}^2 + A^2 = 4$.

In terms of these operators $\mathcal{H}_0$ acquires the form

$$\mathcal{H}_0 = \frac{1}{2}\left(E_T\mathbf{S}^2 + E_S\mathbf{R}^2\right) + Q(\hat{N} - 2)^2, \tag{7}$$

and

$$\mathcal{H}_0 = \frac{1}{2}\left(E_T\mathbf{S}^2 + E_S\mathbf{R}^2 + E_{E_r}\mathbf{R}_1^2\right) + Q(\hat{N} - 2)^2. \tag{8}$$

for $SO(4)$ and $SO(5)$ group, respectively. The last terms in (7) and (8) control the number of electrons given by the operator $\hat{N}$ in DQD.

As is seen from this equation, spin is still conserved in isolated DQD. However, a tunnel contact with metallic leads breaks the spin conservation and reveals the dynamical symmetry of DQD. The mechanism of this non-conservation is *electron cotunneling* with spin flips, when an electron with spin σ enters DQD, whereas another electron with spin σ' leaves it. This process is known to be a source of Kondo effect in tunnel barriers and quantum dots [3]. Eliminating charge degrees of freedom by means of the Schrieffer-Wolff transformation, one usually comes to an exchange-like cotunneling Hamiltonian of the type $J_{cot}\mathbf{S}\cdot\mathbf{s}$, where $J_{cot} \sim W^2$, and W is a lead-dot tunneling amplitude.

Since $E_T - E_S = \delta > 0$, the Kondo effect seems to be irrelevant in DQD with even occupation. However, one should remember that the tunneling W induces additional contribution into indirect exchange between two wells in DQD. As is shown in Refs. [2] this contribution may change the sign of δ provided the excitation E_r is soft enough, but the condition $V/(E_T - E_S) \ll 1$ is still valid. Then the exciton E_r is eliminated from the manifold, the symmetry of DQD is reduced from $SO(5)$ to $SO(4)$ and the Schrieffer-Wolff transformation yields the effective spin Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + J_{cot}^T\mathbf{S}\cdot\mathbf{s} + J_{cot}^{ST}\mathbf{R}\cdot\mathbf{s}, \tag{9}$$

where J_{cot}^{ST} and J_{cot}^{ST} are two indirect exchange coupling parameters which are renormalized by Kondo screening. This screening is given by both vectors **S** and **R**.

The problem of Kondo tunneling within the framework of the Hamiltonian (9) is solved already, at least in the weak coupling limit (see [2] and references therein), so we do not enter the details here. For our further purposes it is important, that this example demonstrates how the dynamical symmetry of S/T pair is revealed in interaction with continuum, which breaks the rotational symmetry of isolated spin system. This interaction inserts its own energy scale $\mathcal{E}$ in the problem (the Kondo temperature T_K in example considered above), and the dynamical symmetry of spin rotator becomes relevant when the T/S energy splitting is comparable with T_K. In more complicated quantum dots the spin manifolds consist of several S/T pairs, and the dynamical symmetry of such dots is described by $SO(n)$ groups (see Ref. [4] where the cases of $n = 5, 7$ are described).

3. From spin rotator to spin ladder

Being armed by the above mathematical tools, we see that any rung of a two-leg spin ladder (Fig. 2a) possesses the same $SO(4)$ symmetry, because two spins 1/2 form a S/T manifold. Therefore the dynamical symmetry is an intrinsic property of spin ladders and decorated spin chains shown in Figs 2b,d. Here we derive the family of Hamiltonians for these systems and discuss various manifestations of this symmetry in their energy spectrum.

Generic Hamiltonian for spin systems under consideration is the Heisenberg-type spin 1/2 ladder Hamiltonian

$$H^{(SL)} = J_t \sum_{\langle i1,i2\rangle} \mathbf{s}_{i1} \cdot \mathbf{s}_{i2} + J_l \sum_{\alpha} \sum_{\langle i\alpha,j\alpha\rangle} \mathbf{s}_{i\alpha} \cdot \mathbf{s}_{j\alpha} \tag{10}$$

Here index $\alpha = 1, 2$ enumerates the legs of the ladder, and the sites $\langle i1, i2\rangle$ belong to the same rung (Fig.2a).

A chain of dimers of localized spins illustrated by Fig. 2b is described by the simplified version of this Hamiltonian

$$H^{SRC} = J_t \sum_{\langle i1,i2\rangle} \mathbf{s}_{i1} \cdot \mathbf{s}_{i2} + J_l \sum_{\langle ij\rangle} \mathbf{s}_{i1} \cdot \mathbf{s}_{j1} \tag{11}$$

The geometry of alternate rungs is chosen in the system (11) to avoid exchange interaction between spins $\mathbf{s}_{i2}$ and $\mathbf{s}_{j2}$.

The transverse coupling may emerge either from direct exchange (in case of localized spins) or from indirect Anderson-type exchange induced by tunneling (similarly to the case of quantum dots). In the latter case the sign of J_t is antiferromagnetic (AFM), in the former case it may be ferromagnetic (FM) as well. The same is valid for J_l.

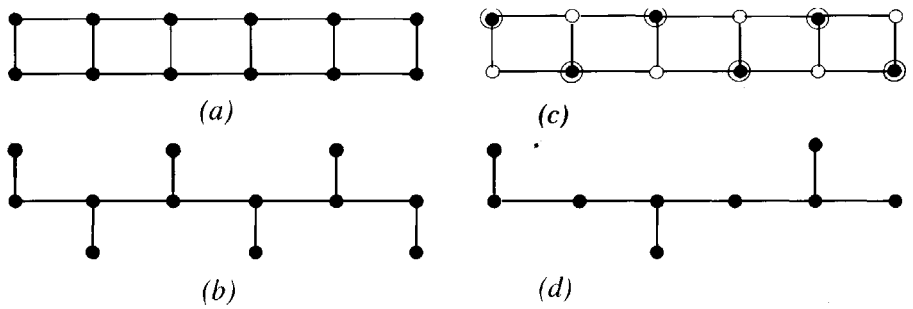

Figure 2. Spin Ladder (a), Spin Rotator Chain (b), Spin ladder in the CDW phase (c) and Alternate Spin Rotator Chain (d).

We start with diagonalization of the Hamiltonian of perpendicularly aligned dimer (cf. Ref. [5]). The $SO(4)$ symmetry stems from the obvious fact that the spin spectrum of a dimer $\{i1, i2\}$ is formed by the same S/T pair as the spin spectrum of DQD studied in the previous section. This analogy prompts us a canonical transformation connecting two pairs of spin vectors, $\{\mathbf{s}_{i1}, \mathbf{s}_{i2}\}$ and $\{\mathbf{S}_i, \mathbf{R}_i\}$: Two sets of spin operators are connected by a simple rotation

$$\mathbf{s}_{i1} = \frac{\mathbf{S}_i + \mathbf{R}_i}{2}, \quad \mathbf{s}_{i2} = \frac{\mathbf{S}_i - \mathbf{R}_i}{2}, \tag{12}$$

Then the Hamiltonian $\mathcal{H}_i$ of a single dimer i is the same as the Hamiltonian (7) of DQD. The total spin of a dimer is not conserved in a spin chain, so the dynamical symmetry of an individual rung is revealed by the modes propagating along the chain [5]. Applying the rotation operation (12) to the Hamiltonians (10) and (11), we transform them to a form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{int} \tag{13}$$

Here $\mathcal{H}_0 = \sum_i \mathcal{H}_i$ is common for both models. It is useful to include the Zeeman term in $\mathcal{H}_i$,

$$\mathcal{H}_i = \frac{1}{2}\left(E_S \mathbf{R}_i^2 + E_T \mathbf{S}_i^2\right) + hS_{iz}. \tag{14}$$

We confine ourselves by a charge sector $N_i = 2$ and omit the Coulomb blockade term for the sake of brevity. The interaction part of SL Hamiltonian transforms under the rotation (12) to the following expression

$$\mathcal{H}_{int}^{SL} = \frac{1}{4} J_l \sum_{\langle ij \rangle} (\mathbf{S}_i \mathbf{S}_j + \mathbf{R}_i \mathbf{R}_j) \tag{15}$$

The interaction part of the SRC Hamiltonian is

$$\mathcal{H}_{int}^{SRC} = \frac{1}{4} J_l \sum_{\langle ij \rangle} (\mathbf{S}_i \mathbf{S}_j + 2\mathbf{R}_i \mathbf{S}_j + \mathbf{R}_i \mathbf{R}_j) \tag{16}$$

One may also consider the *alternate SRC model* (ASRC, see Fig. 2(c)). Its interaction Hamiltonian acquires after rotation (12) the form

$$\mathcal{H}_{int}^{ASRC} = \frac{1}{4} J_l \sum_{\langle ij \rangle} (\mathbf{S}_i \mathbf{S}_j + \mathbf{S}_i \mathbf{R}_j) \tag{17}$$

Now we see that all three effective Hamiltonians belong to the same family. In all cases initial ladder or "semi-ladder" Hamiltonian is transformed into really one-dimensional spin-chain Hamiltonian, which, however, takes into account the hidden symmetry of a dimer. The effective Hamiltonians (15), (16), (17) contain operators $\mathbf{R}$ describing the dynamical symmetry of dimers. The dynamical symmetry turns the spectrum of this Hamiltonians to be richer than that of a standard Heisenberg chain. Like in many other cases, rotation transformation eliminates the antisymmetric combination of two generators.

The transformation (12) reveals the hidden symmetry of spin 1/2 ladder (15). It maps the ladder Hamiltonian onto a pair of coupled chain Hamiltonians: one is the conventional spin 1 chain, the other is a pseudospin chain. Spin $\mathbf{S}_i$ and pseudospin $\mathbf{R}_i$ are coupled kinematically by the commutation relations and by the local Casimir constraint

$$\mathbf{S}_i^2 + \mathbf{R}_i^2 = 3. \tag{18}$$

It is instructive to compare the Hamiltonian (15) with the effective Hamiltonian of spin 1 chain, which arises after decomposition of spin-one operators into a pair of spin 1/2 operators, $\mathbf{S}_i = \mathbf{s}_i + \mathbf{r}_i$ [6]. This decomposition operation transforms initial Hamiltonian into a form similar to H^{SRC} but for spin-one-half operators $\mathbf{s}_i$, $\mathbf{r}_i$. The difference between two cases is that these effective spins commute (unlike the operators $\mathbf{S}_i$, $\mathbf{R}_i$). In other terms, the difference is that the local symmetry of spin-one chain is $SO(3)$ whereas the local symmetry of SRC is $SO(4)$. The spin rotator chains (16), (17) are in some sense intermediate between spin chains and spin ladders. In this cases the spin-pseudospin symmetry is obviously broken by the cross terms $2\mathbf{S}_i\mathbf{R}_j$.

The excitation spectrum of spin ladders may be calculated in terms of operators $\mathbf{S}_i$ and $\mathbf{R}_i$. For example, the well known expression for a gap ΔE in the excitation spectrum in the limit of strong transverse exchange $J_t \gg J_l$ for AFM interaction [5] is

$$\Delta E = J_t + \frac{(J_l/4)^2 \sum_{ij,\alpha\beta} \left(\langle T_{ij}^{\alpha\beta} | \mathbf{R}_i \mathbf{R}_j | S_i S_j \right)^2}{(E_T - E_S)} = J_t + \frac{3J_l^2}{8J_t} \tag{19}$$

(here $T_{ij}^{\alpha\beta}$ and S_iS_j stand for possible triplet projections and spin states at the sites i, j, respectively). The singlet-triplet excitations above this gap are given by the dispersion law $\omega(k) = \Delta E + J_l \cos k$.

The Casimir operator (18) transforms to the local constraint $\sum_{\Lambda=\pm,0,s} f^\dagger_\Lambda f_\Lambda = 1$. To fermionize the generators of $SO(5)$ group, one should add two more spin fermions and one more spinless fermion describing transitions to the excitonic state $|E\rangle$.

We start the studies of elementary excitations in SRC with the aniso-tropic XXZ version. of general effective Hamiltonian. The simplest one is the case (17). The problem is reduced to a standard XY-model for spin one half, and the spinon spectrum may be easily obtained either by bosonization or by spinon-type fermionization. In former case one deals with hard-core bosons, and in the latter one the problem is mapped onto the non-interacting incompressible fermions at half-filling.

Next is a more complicated case of XXZ-SRC model (21) specifically on its simplified alternate version, which is obtained from the Hamiltonian (17). The Hamiltonian of this model is

$$H = \frac{1}{4} J_l \sum_{\langle ij \rangle} (S^+_i S^-_j + S^+_i P^-_j + S^-_i P^+_j + \Delta (S^z_i S^z_j + 2 S^z_i P^z_j). \qquad (26)$$

The S-S part of this Hamiltonian describes the S=1 chain, with the Haldane gap in the excitation spectrum (see, e.g.,[9, 10]). The question is, how do the S-P interaction modifies the gap. We consider the case of FM dimers, when the triplet is the ground state. In this case one has one more gap mode, where the gap equals J_t. This mode is coupled to Haldane branch only via S-P exchange terms in (26).

The spin liquid fermionization approach adopted here is a convenient tool for description of Haldane spectrum. Unlike the S=1/2 model, where the spin-liquid state is easily described by global U(1) invariant modes $T_{ij} T_{ji} = \sum_\sigma f^\dagger_{i\sigma} f_{j\sigma}|^2$, in case of S=1, one deals with variables which *effectively break this symmetry.* One can rewrite the Hamiltonian of SRC model with $\Delta = 0$ in a form

$$H = \frac{1}{4} J_l \sum_{ij} \left[\left(f^\dagger_{i1} f_{j1} + f^\dagger_{i\bar{1}} f_{j\bar{1}} \right) \bar{B}^{0S}_j B^{0S}_i + (f^\dagger_{i\bar{1}} f^\dagger_{j1} C^{0S}_j B^{0S}_i + H.c.) \right] \qquad (27)$$

where $B^{0S}_j = f_{0j} + f_{Sj}$, $C^{0S}_j = f_{0j} - f_{Sj}$. The terms in the first line of Eq. (27) describe coherent propagation of spin fermions accompanied by a backflow on neutral fermions, whereas the terms in the second line are "anomalous" (they do not conserve spin fermion number). For example the propagator $\langle S^+_i S^-_j \rangle$ contains anomalous components $f^\dagger_{i1} f^\dagger_{j\bar{1}} f_{j0} f_{i0} \to F^*_{ij,1\bar{1}} F_{ji,00}$ along with normal ones $f^\dagger_{i1} f_{j1} f^\dagger_{j0} f_{i0}$. Here $F_{ij,\Lambda\Lambda'} = f_{j\Lambda} f_{i\Lambda'}$. The first term in (27) describes the kinetic energy spinon excitations, and two last anomalous term breaking U(1) symmetry are responsible for the Haldane gap.

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and as gauge symmetry breaking terms in the second line. In spin 1 ladder the counterflow term $\sim f^{\dagger}_{i0} f_{j0}$ predetermines the width of spinon band described by the first line of Eq. (27). Apparently, the one extra channel (tripet/singlet transitions in B^{0S}) enhances this effect, because in this case the local constraint imposes further restrictions of phase fluctuations.

The gap itself is due to anomalous correlations described by the second line of Eq. (27). Here the appearance of second channel of spinless excitations results in formation of even and odd operators B^{0S}_j and C^{0S}_j. The Haldane gap closes when the $|0\rangle$ and $|S\rangle$ states are degenerate (the odd operator C^{0S}_j nullifies the anomalous terms responsible for its formation). This means that appearance of $0S$ channel *favors* closing of the Haldane gap.

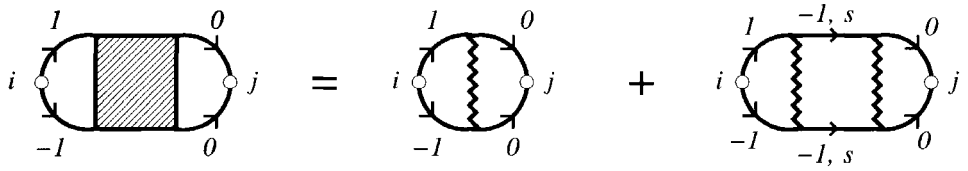

Figure 3. Lowest order contributions to anomalous propagator (27).

In a strong coupling case of $J_t \gg J_l$ both trends above may be considered at least in the lowest order of perturbation theory. In case of spin ladders [5] the 1-st and 2-nd-order in $g = J_l/J_t$ anomalous diagrams are represented in Fig.3.

5. SO(n) dynamical symmetries for a two-leg quantum ladder

It was mentioned in Section II that the dynamical symmetry of DQD becomes $SO(5)$, if charge transfer excitonic state is involved (see Eq. 8). In this section we discuss the origin of this symmetry in spin ladders. This problem arose in a context of $SO(5)$ symmetric $t-J$ model of 2D cuprate superconductors [11]. Later on the version of this theory was formulated for cuprate two-leg ladders [12]. Here we show that the *dynamical* $SO(5)$ group arises in description of Heisenberg ladder, but excitonic states are involved in this symmetry instead of Cooper states.

Let us consider a two-leg quantum ladder depicted in Fig.2a under condition of strong Coulomb blockade imposed on each rung i. We allow electron tunneling t^{α}_{ij} along both legs. This tunneling is described by the Hamiltonian

$$\mathcal{H}_{tun} = \sum_{ij} \sum_{\alpha\sigma} t^{\alpha} d^{\dagger}_{i\alpha\sigma} d_{j\alpha\sigma}. \tag{28}$$

(only nearest-neighbor hopping along the leg is allowed). This hopping results in the appearance of *charged* rungs because each hopping act creates a hole on a

rung j and an electron on a rung i. To treat this charging properly the Coulomb blockade term in the Hamiltonian $\mathcal{H}_i$ (14) should be restored (see Eq. 8), and the terms with excess electron and excess hole should be added. It is more convenient to represent the Hamiltonian $\mathcal{H}_i$ of an individual rung i in terms of diagonal Hubbard operators [see (1)]

$$\mathcal{H}_i = \sum_i \left[\sum_\Lambda E_\Lambda X_i^{\Lambda\Lambda} + \sum_\gamma E_\gamma X_i^{\gamma\gamma} + \sum_\Gamma E_\Gamma X_i^{\Gamma\Gamma} \right] \tag{29}$$

Here the index $\gamma = \alpha\sigma$ stands for the states with one electron with spin σ on a site $i\alpha$ of the rung i, the index $\Gamma = \alpha\sigma$ stands for three-electron states of a rung, where two electrons occupy site $i\alpha$ and one electron with spin σ is located in a site $i\bar{\alpha}$ ($\bar{\alpha} = 2$ if $\alpha = 1$ and v.v). The energy levels E_γ and E_Γ are separated by a Coulomb blockade gap $\sim Q$ from the two-electron states E_Λ. The Hamiltonian (28) in these terms is

$$\mathcal{H}_{tun} = \sum_{ij,\alpha} \sum_{\gamma\Gamma\Lambda} t^\alpha X_i^{\Gamma\Lambda} X_j^{\gamma\Lambda} + H.c. \tag{30}$$

It is seen from (30) that the intersite hopping "charges" two neighboring rungs in a ladder, which was initially neutral, and one should pay the energy $\sim Q$ for each hopping act, like in the generic Hubbard model at half-filling. This energy loss is reduced if an electron-hole pair is created at a given rung i. In this case the electron-hole attraction $V < 0$ partially compensates charging energy Q. Let us assume the hierarchy $Q \gg Q - |V| \gg t$. Then the states $|\Gamma\rangle$ may be excluded from the manifold in favor of excitonic states $|iE_\alpha\rangle$ similar to the states $|E_r\rangle$ introduced in (5). Here $\alpha = 1(2)$ for the electron occupying site $i1(i2)$. If the ground state of a rung is singlet, $|iS\rangle$, then electron and hole have antiparallel spins and the excitation energy is $Q' = Q - |V|$. Even combination of two states $|iE_{(1,2)}\rangle$ form a singlet exciton $|iE\rangle$. Such exciton can propagate coherently along the ladder unlike single electron, whose tunneling leaves a trace of charged states according to (30). Indeed, translation of e-h pair from a rung i to a neighboring rung $i+1$ can be presented as coherent tunneling of electron from a site $i\alpha$ to a site $i+1, \alpha$ and another electron in the opposite direction (from $i+1, \bar{\alpha}$ to $i, \bar{\alpha}$. The exciton propagation is described by the following term in effective Hamiltonian:

$$H_{ex} = \sum_i K^S X_i^{SE} X_{i\pm1}^{ES} \tag{31}$$

with effective exchange coupling constant $K^S = |t_1 t_2|/Q'$, and the dispersion law describing coherent exciton propagation is $\epsilon_S(k) = 2K^S \cos k$. As was shown in Section 2, the manifold $\{iS, iT, iE\}$ possesses the local dynamical symmetry $SO(5)$ [see Eqs. (5), (6)], and this symmetry allows existence of

coherent collective singlet exciton mode. The Hamiltonian (31) acquires a form $H_{ex} = (K^S/4)\sum_{ij}\widetilde{A}_i\widetilde{A}_j$ in terms of generators of SO(5) group (5), where $\widetilde{A}_i = (\mathbf{R}_i^2 - 1)A_i$. There is one more collective mode, namely triplet exciton $|E_\mu\rangle$ ($\mu = \pm 1, 0$) separated by the gap $\sim J_t$ from the singlet exciton. In case of triplet ground state ($J_t < 0$), this mode becomes the lowest one, and the Hamiltonian similar to (31) may be derived for triplet exciton propagation with operators $X_i^{TE_\mu}$ replacing X_i^{SE}. In this case the manifold $\{iS, iT, iE_\mu\}$ consists of one singlet and two triplets, and the corresponding dynamical group is $SO(7)$ [4].

If exchange and excitonic gaps are comparable in magnitude, then the interplay between exciton and magnon modes is possible, and dynamical symmetry will result in observable physical effects. Like in cuprate ladder, [13], the excitonic instability can develop for certain values of model parameters, which results in phase separation and, in particular in formation of CDW phase illustrated by Fig. 2c (where double and empty circles stand for doubly occupied and empty sites respectively).

6. Concluding remarks

We rederived a family of Hamiltonians for quantum dots and quantum ladders in terms of $SO(4)$ group, which describes the dynamical symmetry of a spin rotator [2]. We exploited the fact that in case, when the Hamiltonian $\mathcal{H}$ contains blocks $\mathcal{H}_i$ formed by two sites occupied by spins 1/2, one may use its eigenstates (singlet-triplet manifolds) as a basis for representing the spin invariants entering $\mathcal{H}$. These invariants contain the Runge-Lenz-like vectors $\mathbf{R}_i$ along with the usual spin vectors $\mathbf{S}_i$. If the electron-hole pairs are also included in the set of eigenstates, then the local dynamical symmetry of $\mathcal{H}_i$ is characterized by the $SO(n)$ group with $n = 5$ or 7 for a singlet and triplet ground state of $\mathcal{H}_i$, respectively. The elementary excitations in quantum dots and quantum ladders are described by means of generators of $SO(n)$ groups and the interplay between different branches of excitation spectra is a direct manifestation of local dynamical symmetry violated by non-local interactions.

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