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Spin gap in a spiral staircase model

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Abstract

We investigate the formation of spin gap in one-dimensional models characterized by the groups with hidden symmetries. We introduce a new class of Hamiltonians for description of spin staircases—the spin systems intermediate between 2-leg ladders and $S = 1$ spin chains. The spin exchange anisotropy along legs is described by the angle of spiral twist. The properties of a special case of spin rotator chain (SRC) corresponding to a flat 1-leg ladder is considered by means of fermionization approach based on Jordan–Wigner transformation. The influence of dynamical hidden symmetries on the scaling properties of the spin gap is discussed.

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Haldane's conjecture [1] says that the properties of SU(2) spin S Heisenberg antiferromagnetic (HAF) spin chains are different for integer and half-integer spins. The excitation in the HAF chains with half-integer spins are gapless [2], whereas for the integer spin there exists a gap. There is an important mapping between $s = \frac{1}{2}$ HAF chains and Luttinger liquids [3] which allows to treat such chains by means of fermionization and bosonization methods. However $S = 1$ chains represent more complex object where Haldane conjecture still remains a hypothesis [4]. In this paper we report a new class of three-parameter models described by dynamical groups in order to demonstrate the formation of spin gap in systems characterized by symmetries different from SU(2). We call this family spin staircase model (SSM). The SSM is introduced to include conventional 2-leg ladders and describe realizations intermediate between ladders and chains.

The SSM represents two spin $s = \frac{1}{2}$ chains with antiferromagnetic (AF) interaction along legs and

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ferromagnetic (FM) coupling within a rung. The Hamiltonian is given by

$$H = J_{\parallel} \sum_i \left(\vec{s}_{1,i}\vec{s}_{1,i+1} + \cos^2(\theta/2)\vec{s}_{2,i}\vec{s}_{2,i+1}\right) - J_{\perp}\sum_i \vec{s}_{1i}\vec{s}_{2i} - \sum_i \sum_{\alpha=1,2} \vec{h}\vec{s}_{\alpha,i}. \quad (1)$$

This model may be interpreted as a result of twist deformation of 2-leg ladder while twist is performed around one of the legs. Such a spiral structure is characterized by the angle θ. Two limiting cases $\theta = 0$ and π, correspond to standard 2-leg ladder with isotropic exchange and spin-rotator chain (SRC) [5,6] model, respectively. In a latter case the interaction between spins sitting at the ends of different rungs is assumed to be negligibly small compared to both the interaction along the rung and the leg. The model is a natural extension of the $S = 1$ chain to a case where the states on a given rung form a triplet/singlet pair. It is convenient to reformulate model (1) in terms of new variables defined on the rung $\vec{S}_i = \vec{s}_{1,i} + \vec{s}_{2,i}$, $\vec{R}_i = \vec{s}_{1,i} - \vec{s}_{2,i}$, $\vec{S}_i$ stands for a triplet $S = 1$ ground state and singlet $S = 0$ excited state:

$$H = \sum_i \left[\frac{J_{\parallel}}{4}\left\{(\vec{S}_i\vec{S}_{i+1} + \vec{R}_i\vec{R}_{i+1})\left(1 + \cos^2\left(\frac{\theta}{2}\right)\right) + \sin^2\left(\frac{\theta}{2}\right)(\vec{S}_i\vec{R}_{i+1} + \vec{R}_i\vec{S}_{i+1})\right\} - \frac{J_{\perp}}{4}(\vec{S}_i^2 - \vec{R}_i^2)\right].$$

The set of operators $\vec{S}_i, \vec{R}_i$ fully defines the o_4 algebra through the following commutation relations:

$$[S_i^{\alpha}, S_j^{\beta}] = \mathrm{i}\delta_{ij}\varepsilon_{\alpha\beta\gamma}S_i^{\gamma}, \quad [R_i^{\alpha}, R_j^{\beta}] = \mathrm{i}\delta_{ij}\varepsilon_{\alpha\beta\gamma}S_i^{\gamma},$$
$$[R_i^{\alpha}, S_j^{\beta}] = \mathrm{i}\delta_{ij}\varepsilon_{\alpha\beta\gamma}R_i^{\gamma}, \quad (2)$$

while the Casimir operators are $(\vec{S}_i)^2 + (\vec{R}_i)^2 = 3$, $(\vec{S}_i \cdot \vec{R}_i) = 0$. Here we confine ourselves by the SRC model $\theta = \pi$. While isotropic models (1) respect SO(4) symmetry of $\vec{S}_i, \vec{R}_i$ operators, the generalization of Eq. (1) for the cases of easy axis/easy plane anisotropy

$$H_{\parallel} = \frac{J_{\parallel}^x}{8}\sum_i [S_i^+ S_{i+1}^- + S_i^+ R_{i+1}^- + (S \leftrightarrow R) + \mathrm{h.c.}] + \frac{J_{\parallel}^z}{4}\sum_i [S_i^z S_{i+1}^z + S_i^z R_{i+1}^z + (S^z \leftrightarrow R^z)], \quad (3)$$

$$H_{\perp}^i = \frac{J_{\perp}^x}{8}((R_i^x)^2 + (R_i^y)^2) + \frac{J_{\perp}^z}{4}(R_i^z)^2 - (\vec{R}_i \leftrightarrow \vec{S}_i)$$

corresponds to the case of symmetry reduction from SO(4) to SO(2) × SO(2) × Z_2 × Z_2.

Models (1,3) admit fermionization by means of SO(4) Jordan–Wigner transformation [6,7] written in terms of unconstrained spinor field $(f_{\uparrow} f_{\downarrow})$ as follows:

$$S_j^+ = \sqrt{2}\left(f_{\uparrow j}^{\dagger}(1 - n_{\downarrow j})K_j + K_j^{\dagger} f_{\downarrow j}(1 - n_{\uparrow j})\right),$$
$$S_j^- = (S_j^+)^{\dagger}, \quad S_j^z = n_{\uparrow j} - n_{\downarrow j}, \quad (4)$$

$$R_j^+ = \sqrt{2}\left(f_{\uparrow j}^{\dagger} n_{\downarrow j} K_j + K_j^{\dagger} f_{\downarrow j} n_{\uparrow j}\right),$$
$$R_j^- = (R_j^+)^{\dagger}, \quad R_j^z = f_{\uparrow j}^{\dagger} f_{\downarrow j}^{\dagger} + f_{\downarrow j} f_{\uparrow j}, \quad (5)$$

where $n_{\sigma j} = f_{\sigma j}^{\dagger} f_{\sigma j}$ and the string operator is given by

$$K_j = \exp\left[\mathrm{i}\pi \sum_{k<j,\sigma} n_{\sigma k}\right] = \prod_{k<j}(1 - 2n_{\uparrow k})(1 - 2n_{\downarrow k}). \quad (6)$$

Models (1,3) possess special hidden symmetries Z_2 and $Z_2 \times Z_2$ [6] connected with discrete transformations in a 6-D space of SO(4) group generators. These hidden symmetries influence the scaling properties of the spin gap and determine the order parameter [6].

Let us consider easy plane SRC model. Hamiltonian (3) written in terms of new fermionic variables $a = (f_{\uparrow} + f_{\downarrow}^{\dagger})/\sqrt{2}$, $b = (f_{\downarrow}^{\dagger} - f_{\uparrow})/\sqrt{2}$ is given by

$$H_{\parallel} = J_{\parallel}^x \sum_i \left(a_i^{\dagger} a_{i+1} + a_{i+1}^{\dagger} a_i\right)\cos(\pi n_i^b) + J_{\parallel}^z \sum_i \left(n_i^a - \tfrac{1}{2}\right)\left(n_{i+1}^a - \tfrac{1}{2}\right) \quad (7)$$

and $H_\perp = \sum_i H_\perp^i$ with

$$H_\perp^i = -\frac{J_\perp^x}{2}\left(a_i^\dagger b_i + b_i^\dagger a_i\right) - J_\perp^z\left(n_i^a - \tfrac{1}{2}\right)\left(n_i^b - \tfrac{1}{2}\right), \tag{8}$$

where the shorthand notations $n^a = a^\dagger a$, $n^b = b^\dagger b$ and $\cos(\pi n^b) = \mathrm{Re}\,\exp(\pm i\pi n^b) = 1 - 2n^b$ are used. The Hamiltonian (7,8) represent the key result of this paper. It establishes the mapping between SO(4) SRC chain and effective $s=\frac{1}{2}$ fermionic models. The effective interaction in equivalent $s=\frac{1}{2}$ model consists of Hubbard-like exchange and kinematic interaction given by cosine factor in Eq. (7). Fermions b do not have any dynamics. The main advantage (7,8) is that unlike e.g. t–J_z models [7], this interaction describes 4-fermion scattering and can be treated both by renormalization group and bosonization approaches [6].

The continuum representation for spin operators $\vec{s}_a, \vec{s}_b$ in Abelian Bosonization formalism reads

$$s_i^{\pm}(x) \sim e^{\pm i\theta_i}(\cos(\pi x) + \cos(2\phi_i)),$$
$$s_i^z(x) \sim \pi^{-1}\partial_x\phi_i + \cos(\pi x + 2\phi_i) \tag{9}$$

with canonically conjugated variables ϕ_i and $\Pi_i = \partial_x\theta_i$ $(i = a, b)$. The refined analysis based on two-stage renormalization procedure shows that the easy plane SRC model possess a spin gap (Fig. 1) which scales as $\Delta \sim J_\parallel (J_\perp/J_\parallel)^{2/3}$. This fractional scaling law is different from those for conventional 2-leg ladders where $\Delta \sim J_\perp$ [8]. The formation of the gap is determined by the backward scattering processes of the field a on the random potential associated with fluctuations of $\cos\theta_a$. The fully isotropic case, $J_\perp^x = J_\perp^z = J_\perp$ yields the same estimate $\Delta \sim J_\parallel^{1/3}(J_\perp)^{2/3}$.

The case $J_\perp \gg J_\parallel$ correspond to Haldane $S=1$ gap and requires more careful analysis.

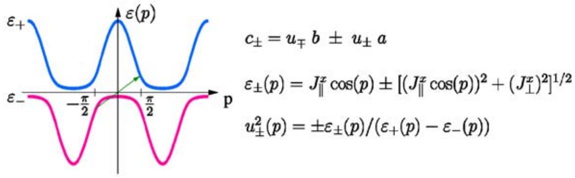

Fig. 1. Dispersion law for hybridized spin fermions. Arrow indicates backward scattering processes responsible for renormalization of spin gap $\Delta_0 \sim (J_\perp^x)^2/J_\parallel \to \Delta \sim (J_\perp^x)^\gamma (J_\parallel^x)^{1-\gamma}$.

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